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Book Review

Ferenc Forgó, Jenö Szép and Ferenc Szidarovszky: *Introduction to the Theory of Games: Concepts, Methods, Applications.* Series: Nonconvex Optimization and its Applications, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1999, 339 pages, ISBN 0-7923-5775-2.

As the authors mention, this book is the outcome of its previous edition *Introduction to the Theory of Games*, Akadémia Kiadó Budapest and D. Reidel, Dordrecht, 1985, and material from the graduate courses that the authors have taught for several years. In general terms, game theory, which has been founded by von Neuman and Morgenstern, studies situations of conflict of interest between two or more parties, where each party strives to achieve the best outcome in his own respect and by possibly forming coalitions with other parties. Based on an axiomatic definition of rationality among the players, the objective is to predict the outcome of the conflict. The book is divided into two parts, one dealing with noncooperative games and one with cooperative games, with problems at the end of each part.

The first part of the book begins with an introductory chapter were the extensive form and the normal or strategic form of representing a game are presented. In the normal or strategic form a game is represented by n players where each player $i = 1, \dots, n$ has a set of possible strategies S_i, and payoff functions $f_i : S_1 \times \dots \times S_n \to \mathcal{R}$ which assign a real value (some type of gain, monetary or otherwise defined) for each combination of strategies of all players. Through the definition of the extensive form, the authors also introduce formally notions such as games of complete and incomplete information, perfect information and those of perfect and imperfect recall. The important definition of a strategy in a game is also formally presented. Chapters 2–4 could might as well have constituted one chapter, since the topic of discussion is that of Nash equilibria. A game, being a situation of conflict, can also be viewed as an interaction of forces in a system where depending on their nature can lead the system into different states. The concept of an equilibrium state is that state of the system, where balance between the forces occurs and any variation of any of the forces will disturb the balance. This notion of an equilibrium state as applied to games was introduced by John Nash in 1950, where the forces are essentially the conflicting interests of the individual players and their nature is defined by the strategies which the players follow. A Nash equilibrium point in a game, is a combination of strategies of the players such that it is of no interest to any of the players to change its strategy unilaterally. In other words a strategies tuple $(s_1^*, \ldots, s_n^*) \in S_1 \times \cdots \times S_n$ where

$$f_i(s_1^*, \ldots, s_i^*, \ldots, s_n^*) \geqslant f_i(s_1^*, \ldots, s_i, \ldots, s_n^*), \quad \forall i, s_i \in S_i.$$

Under the assumption that all players are rational (in the sense that they strive for their own interest) an equilibrium point of a game can be considered as the *solution* to the game, or the way the players will play. In Chapter 2 the authors define Nash equilibria and show that they are equivalent to certain axioms regarding rational behavior and consistency among games. Chapter 3 provides existence theorems for Nash equilibria by imposing the condition of convexity in the strategy sets of the players using the fixed point theorem of Brouwer and its generalizations. In Chapter 4 more restrictive conditions are mentioned, such that uniqueness of a Nash equilibrium is established, and the authors also present an algorithm for finding the equilibrium. There is a simple way to always satisfy the convexity condition on the strategy sets by creating new strategy sets $S_i^{'}$ which will consist of all the convex combinations of the elements of S_i for each player i. Modifying respectively the payoff functions we then have a game for which there will always exist a Nash equilibrium, and this is the topic which the authors present in Chapters 5 and 6, together with an algorithm by Scarf and Hansen for computing it. Having established a adequate theoretical framework for noncooperative games in the first six chapters of the book, the authors examine in the next seven remaining chapters of part one special types of games such as the oligopoly game, two-person zero-sum games, matrix games, games over the unit hypercube, bimatrix games, repeated games and games with incomplete information. In each chapter key theoretical results on the respective topic are presented, such as minmax theorems in the two-person zero-sum games and the relationship between matrix games and linear programming.

Cooperative games are treated in the second part of the book. If we have a set of players $N = \{1, 2, \ldots, n\}$ in a game which we allow coalitions or formations of groups of players, we can have $2^n - 1$ possible coalitions, which is the size of the power set of N excluding the empty set. We define also a *characteristic function* $v(S)$ for each coalition $S \subset N$, which assigns a maximum payoff or monetary value to coalition S given minimum payoff of the players not in the coalition. Now instead of equilibria, solutions to these type of games are ways of *distributing* the total payoff among the players in the coalition. Various solutions or ways of wealth distribution are presented by the authors in Chapters 15 through 19 and namely the core, the stable set, the nucleuolus, the Shapley value and the kernel. Finally various applications and extensions of cooperative games are presented in the last four chapters of the book such as the case where there is no homogeneity of utilities for the payoffs among the players.

My immediate impression of the book is that it does not fit in the realm of classical textbooks. This is not a consequence of its context but rather of its presentation of the material covered which lacks exposition, regardless of the various examples and exercise problems included. Notably, with the exception of Chapters 7 and 9, the rest of the chapters are short with a maximum of 16 pages total and usually less than that. However what the book lacks in exposition and insights is greatly compensated from the plethora of key theoretical results for most aspects of contemporary game theory it covers. In this aspect it fullfils the purpose of the book as a text in a graduate course on game theory, since the initiated reader along with the insights provided by the instructor would build a solid theoretical foundation on the subject. Moreover, this book can also be of value to researchers on the field as a good reference text.

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